

Evaluation of Resampling Methods in Multigroup Truncated Spline Semiparametric Path Analysis: A Simulation Study of Artificial Intelligence in Student Learning

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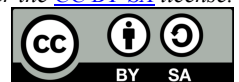
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ABSTRACT

This study aims to evaluate the performance of group-specific resampling in a truncated spline-based semiparametric multigroup path analysis model with heterogeneous relationship structures across groups. The study used a simulation approach based on empirical patterns in AI Literacy data. The multigroup concept is represented by the interaction of dummy variables within a single semiparametric model, thereby allowing for differences in the structures of linear and nonlinear relationships across data groups. The evaluation was conducted across four resampling method combinations: bootstrap-bootstrap, jackknife-jackknife, bootstrap-jackknife, and jackknife-bootstrap. Method performance was evaluated using the average bias, average standard error, standard error ratio, and coefficient of determination (R^2). The results indicate that all method combinations yield relatively consistent average bias values. However, the jackknife-jackknife combination produces the smallest average standard error and standard error ratio, thereby providing better inference stability compared to the other combinations. The results also indicate that the jackknife method tends to be more adaptive for nonlinear relationships based on truncated splines. Thus, the selection of resampling methods for semiparametric multigroup models must consider the characteristics of the relationships within each data group to yield more stable and accurate inferences.

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1. INTRODUCTION

Path analysis was first introduced by Sewall Wright, a geneticist, in 1934 as a method for examining direct and indirect effects between variables [1]. Conventional path analysis generally uses a parametric approach that assumes a linear relationship between variables. This assumption often becomes a limitation when empirical data show relationship patterns that are not entirely linear or exhibit changes at certain intervals, without information about the known form of the relationship between variables.

When the assumption of linearity is violated, conventional path analysis is invalid and can produce biased parameter estimates. The model used to analyze relationships between variables must reflect the actual data patterns [2]. If the linearity assumption is not met and no information on existing relationship patterns is available, parametric path parameter estimation cannot be performed, so a nonparametric approach is used. In situations where the relationship patterns of

some variables are known, and those of others are unknown, semiparametric path analysis can combine parametric and nonparametric methods [3].

The linearity assumption was tested, and the form of the relationship between variables was determined using the Ramsey Regression Specification Error Test (RESET). Ramsey RESET is used to assess whether the relationship between an exogenous variable and other variables is linear or exhibits a nonlinear pattern [4]. Along with the development of analytical methods, modifications to the Ramsey RESET have been extensively developed to detect more complex forms of nonlinear relationships. One such development was carried out by Hidayat et al., who modified the Ramsey RESET to determine the form of nonparametric relationships between variables in a semiparametric model. This approach allows identification of the spline order and determination of the optimal number of knots in nonparametric relationships, making the resulting model more flexible and capable of representing data patterns more accurately [5].

Semiparametric path analysis subsequently evolved into a multigroup semiparametric model for data consisting of multiple observation groups. The multigroup approach is used when there is a possibility that the relationships between variables differ across groups, such that a single model is deemed incapable of accurately representing the overall characteristics of the data. Multigroup semiparametric models allow parameter estimates to differ across groups, thereby capturing heterogeneity in the structure of relationships. This approach is particularly important for empirical data that exhibit differences in social, behavioral, or environmental characteristics across groups.

The semiparametric approach is widely used when the relationships among variables do not fully follow a specific functional form. In such conditions, some variables can be modeled using clear parametric forms, while others are modeled nonparametrically to accommodate relationship patterns that are not known in advance. Therefore, semiparametric models are more flexible in capturing complex relationship patterns than purely parametric models, which require assumptions about the functional form from the outset of the analysis [6]. However, in semiparametric analysis, standard errors cannot be obtained directly as in conventional parametric models; therefore, a resampling approach is required to estimate them [7].

Resampling is a method that repeatedly samples from the original data to construct the empirical distribution of an estimator. Two of the most commonly used resampling methods are the bootstrap and jackknife. The bootstrap method works by randomly resampling from the original data with replacement, whereas the jackknife method is performed by repeatedly resampling without replacement [8], [9].

Various previous studies have discussed the use of bootstrap and jackknife methods in semiparametric models, particularly for parameter estimation and statistical inference. Deng and Wang demonstrated that the bootstrap approach can provide good inferential approximations in semiparametric regression models [10]. Furthermore, the study by You and Zhou also shows that the bootstrap is effective when applied to partial semiparametric models with complex error structures [11]. On the other hand, the study by Hidayat et al. shows that jackknife resampling produces a smaller average bias compared to bootstrap resampling in semiparametric path analysis [7]. The study used a double resampling approach and showed that combining resampling methods can yield different inferential performance compared to a single resampling approach. However, studies specifically evaluating the performance of resampling methods in multigroup semiparametric spline models remain relatively limited. Most previous research has focused on single semiparametric models or on homogeneous conditions across groups, and thus has not been able to capture empirical conditions in which the structure of the relationships differs across groups. On the other hand, heterogeneity in model structure across groups can affect the performance of the resampling method used, both in terms of estimator stability and the accuracy of standard error estimates [12]. Therefore, research is needed that specifically evaluates the effectiveness of different resampling methods for each group within the framework of spline-based semiparametric multigroup path analysis.

The main novelty of this study lies in the evaluation of the performance of group-specific resampling in semiparametric multigroup path analysis models with heterogeneous relationship structures across groups. This study not only considers differences in group characteristics but also

evaluates how different resampling methods affect the stability of parameter estimates and the accuracy of statistical inferences within each group. This approach is expected to provide a methodological contribution to the development of inference in multigroup semiparametric models, particularly in spline-based models that tend to be sensitive to local data variations [13]. Based on this, this study aims to compare the performance of several resampling combinations, namely bootstrap–bootstrap, bootstrap–jackknife, jackknife–bootstrap, and jackknife–jackknife in producing stable standard error estimates in a multigroup truncated spline semiparametric path analysis model.

In this study, the context of artificial intelligence (AI)-based learning was used as the empirical basis for developing simulation scenarios. The use of the AI context was chosen because the phenomenon of AI use in learning exhibits complex patterns of relationships that may vary across groups, making it relevant for representing heterogeneity in a multigroup semiparametric model.

2. RESEARCH METHOD

This study uses a simulation study to evaluate the performance of resampling methods in truncated-spline semiparametric path analysis within a multigroup model. The research data are systematically generated based on predetermined scenarios, allowing the methods to be tested under controlled conditions. The multigroup concept in this study is represented by a dummy variable incorporated into the semiparametric path analysis model. This dummy variable indicates differences among data groups within a given model, allowing each group's characteristics to be represented.

This study does not focus on testing differences in the relationships among variables across groups, but rather on evaluating the performance of resampling methods in estimating the model's standard errors under multigroup data conditions. Furthermore, to estimate the uncertainty of model parameters, resampling methods, including bootstrap and jackknife, are applied across several combination scenarios for each data group. The performance of each resampling scenario is then evaluated using several performance measures, namely average bias, average standard error, standard error ratio, and coefficient of determination, enabling the identification of the resampling method that provides the most stable estimates in the multigroup semiparametric path analysis model.

The path model used in this study consists of two exogenous variables, namely AI Use Attitudes (X_1) and AI Learning Resources (X_2), one intervening variable Academic Adaptation (Y_1), and one endogenous variable AI Literacy (Y_2). The path diagram for this study shown in Figure 1.

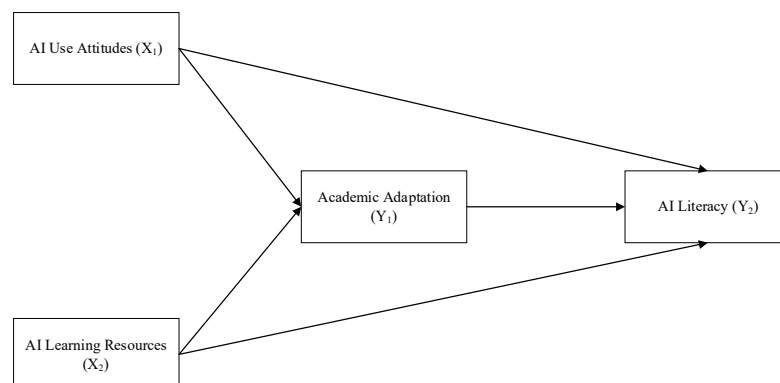


Figure 1. Path Diagram

2. 1. Empirical Data and the Basis for Developing Simulation Scenarios

The data on AI literacy in this study served as the empirical basis for developing simulation data. The patterns of relationships among variables were first identified using the Ramsey RESET test to determine whether the relationships were linear or nonlinear. The results of

this identification were then used as a reference in constructing the simulation data generation structure for each group, so that the resulting simulation data would more realistically represent the characteristics of the relationships in the real data. Subsequently, this simulation data was consistently applied across all resampling combination scenarios, namely bootstrap–bootstrap, bootstrap–jackknife, jackknife–bootstrap, and jackknife–jackknife, to evaluate the performance of resampling methods in the multigroup truncated spline semiparametric path analysis model.

The linearity assumption in this study was tested using the Ramsey RESET method. The linearity test was conducted to determine whether the relationship between variables followed a linear pattern or showed a nonlinear pattern that required a more flexible modeling approach. The linearity test was conducted on two data groups simultaneously. The results of the linearity assumption test are presented in Table 2.

Table 2. Results of Linearity Assumption Testing

Relationship Between Variables	p-value	Group 1	Group 2
AI Use Attitudes (X_1) on Academic Adaption (Y_1)	0.0126	Linear	Truncated Spline Linear 1 Knot
AI Learning Resources (X_2) on Academic Adaption (Y_1)	0.1483	Linear	Linear
AI Use Attitudes (X_1) on AI Literacy (Y_2)	0.0760	Linear	Linear
AI Learning Resources (X_2) on AI Literacy (Y_2)	0.0820	Linear	Linear
Academic Adaption (Y_1) on AI Literacy (Y_2)	0.0103	Linear	Truncated Spline Linear 1 Knot

Based on the results in Table 2, it was found that the relationship between AI Use Attitudes (X_1) on Academic Adaptation (Y_1) and between Academic Adaptation (Y_1) on AI Literacy (Y_2), both had p-values of 0.0126 and 0.0103, indicating that in Group 1, the relationship between the two variables was linear. In Group 2, the relationship was modeled using a truncated spline with 1 knot. Furthermore, the relationship between AI Learning Resources (X_2) on Academic Adaptation (Y_1), AI Use Attitudes (X_1) and AI Learning Resources (X_2) on AI Literacy (Y_2) showed p-values of 0.1483, 0.0760, and 0.0820, respectively, indicating that the relationships in both data groups followed a linear pattern.

Based on these conditions, the modeling results are as follows.

$$\begin{aligned}\hat{f}_{1i} &= \hat{\beta}_{10} + \hat{\beta}_{11}x_{1i} + \hat{\beta}_{12}D_i + \hat{\beta}_{13}x_{1i}D_i + \hat{\beta}_{14}D_i(x_{1i} - k)_+ + \hat{\beta}_{15}x_{2i} + \hat{\beta}_{16}x_{2i}D_i \\ \hat{f}_{2i} &= \hat{\beta}_{20} + \hat{\beta}_{21}x_{1i} + \hat{\beta}_{22}D_i + \hat{\beta}_{23}x_{1i}D_i + \hat{\beta}_{24}x_{2i} + \hat{\beta}_{25}x_{2i}D_i + \hat{\beta}_{26}y_{1i} + \hat{\beta}_{27}y_{1i}D_i \\ &\quad + \hat{\beta}_{28}D_i(y_{1i} - k)_+\end{aligned}$$

The estimated path coefficients, which will serve as the basis for the simulation, are as follows.

$$\begin{aligned}\hat{f}_{1i} &= 2.541 + 0.144x_{1i} + 0.109D_i + 0.028x_{1i}D_i - 0.211D_i(x_{1i} - 2.199)_+ + 0.129x_{2i} \\ &\quad - 0.061x_{2i}D_i \\ \hat{f}_{2i} &= 4.658 - 0.300x_{1i} + 6.150D_i + 0.647x_{1i}D_i - 0.295x_{2i} + 0.726x_{2i}D_i + 0.133y_{1i} \\ &\quad - 3.921y_{1i}D_i + 3.995D_i(y_{1i} - 2.617)_+\end{aligned}$$

With

$$D_i = \begin{cases} 0, & \text{Group 1} \\ 1, & \text{Group 2} \end{cases}$$

The equation for group 1 can be obtained by substituting the value $D_i = 0$, resulting in the following equation.

$$\begin{aligned}\hat{f}_{1i} &= 2.541 + 0.144x_{1i} + 0.129\hat{\beta}_{15}x_{2i} \\ \hat{f}_{2i} &= 4.658 - 0.300x_{1i} - 0.295x_{2i} + 0.133y_{1i}\end{aligned}$$

The equation for group 2 is obtained by substituting the value $D_i = 1$, as follows.

$$\begin{aligned}\hat{f}_{1i} &= 2.650 + 0.172x_{1i} - 0.211(x_{1i} - 2.199)_+ + 0.068x_{2i} \\ \hat{f}_{2i} &= 10.808 + 0.347x_{1i} + 0.431x_{2i} - 3.788y_{1i} + 3.995(y_{1i} - 2.617)_+\end{aligned}$$

The results of the linearity test and the form of the relationship, as indicated by the Ramsey RESET, suggest that Group 1 describes a fully linear relationship, whereas Group 2 represents a semiparametric relationship. These conditions will serve as the basis for generating simulation data across all existing scenarios to assess the resampling method's performance for each group in the multigroup semiparametric path model.

2. 2. Data Generation for the Simulation Study

Data generation for the simulation study in this research was based on the characteristics of the empirical relationships derived from student AI literacy data. The empirical data were first analyzed using the Ramsey RESET test to determine the form of the relationships among the variables. The results of this identification were then used to construct the simulation data generation mechanism, ensuring the generated data more realistically reflects the characteristics of the relationships in the real data. This study used two groups, with a total sample size of 83: 42 in group 1 and 41 in group 2. The multigroup approach used in this study involved interaction with dummy variables.

The simulation model was constructed based on a research path diagram consisting of two exogenous variables, namely AI Use Attitudes (X_1) and AI Learning Resources (X_2), one intervening variable, Academic Adaptation (Y_1), and one endogenous variable, AI Literacy (Y_2). The path coefficients in the model were obtained through empirical data analysis, ensuring that the relationship structure used in the simulation closely follows the patterns observed in the secondary data. The steps of the simulation study conducted in this research are as follows.

1. Set the sample size to $n = 83$.
2. Create a research path diagram, then formulate the appropriate semiparametric multigroup model equations.
3. Analyzing the empirical data to obtain path coefficients and identify linear relationships, as well as truncated linear splines with a single knot point.
4. Calculating the Mean Squared Error (MSE) based on the results of the empirical model estimation. In this study, an MSE of 0.333 was used, obtained from the Literacy AI empirical model.
5. Defining the exogenous variable data based on the empirical research data.
6. Generate error components following a multivariate normal distribution with mean $E(\varepsilon) = 0$ and covarians matrix $Var(\varepsilon) = \Sigma$, assuming homogeneous variance across groups ($\sigma_1^2 = \sigma_2^2$) using an error variance of MSE: 0.333.
7. Forming the intervening variable and endogenous variable data using the multigroup semiparametric model equation, namely

$$Y_{1i} = 2.541 + 0.144x_{1i} + 0.109D_i + 0.028x_{1i}D_i - 0.211D_i(x_{1i} - 2.199)_+ + 0.129x_{2i} - 0.061x_{2i}D_i + \varepsilon_{1i}$$

$$Y_{2i} = 4.658 - 0.300x_{1i} + 6.150D_i + 0.647x_{1i}D_i - 0.295x_{2i} + 0.726x_{2i}D_i + 0.133y_{1i} - 3.921y_{1i}D_i + 3.995D_i(y_{1i} - 2.617)_+ + \varepsilon_{2i}$$
8. Apply group-specific resampling scenarios: Bootstrap-Bootstrap (BB), Jackknife-Jackknife (JJ), Bootstrap-Jackknife (BJ), and Jackknife-Bootstrap (JB).
9. Estimating the model for each resampling result and calculating performance metrics such as average bias, average standard error, standard error ratio, and coefficient of determination (R^2).
10. Repeating the simulation process for the specified number of iterations, 500 times, to obtain stable and representative estimation results for evaluating the performance of the methods used.

2. 3. Group-Specific Resampling Procedure

Resampling methods were applied separately to each data group to preserve the heterogeneous relationships between groups. The combination of resampling methods used consists of: Bootstrap-Bootstrap (BB), Jackknife-Jackknife (JJ), Bootstrap-Jackknife (BJ), and Jackknife-Bootstrap (JB).

In the bootstrap method, sampling is performed with replacement, using a sample size equal to the original data (n). This approach follows the standard bootstrap principle introduced by Efron, in which each bootstrap sample approximates the empirical distribution of the observed data while maintaining the same information size [14]. Sementara itu, metode Jackknife menggunakan pendekatan delete-5% dengan menghapus sebagian observasi pada setiap iterasi resampling

sehingga ukuran sampel menjadi $n^* = 0.95 \times n$. This approach was selected to reduce excessive sensitivity and local instability that frequently occur in semiparametric truncated spline estimation. In spline-based models, removing only a single observation may lead to disproportionate fluctuations around knot regions, whereas delete-d jackknife provides a more stable assessment of estimator robustness while preserving the overall structural characteristics of the data [15] [16].

For each resampling scenario, the simulation process is carried out in the following steps:

1. Generate data based on the relationship structure of each group.
2. Separate the data by group.
3. Apply the resampling method according to the combination of BB, JJ, BJ, and JB.
4. Combine the resampling results of the two groups.
5. Estimating the semiparametric multigroup model.
6. Calculating estimation performance measures, including average bias, average standard error, standard error ratio, and coefficient of determination.

The resampling procedures were applied separately within each group to preserve the heterogeneous structural characteristics represented by the interaction-based multigroup framework. This strategy allows different inferential behaviors to be evaluated under both linear and truncated-spline relationships. The overall group-specific resampling procedure can be illustrated in a diagram such as the one shown in Figure 2.

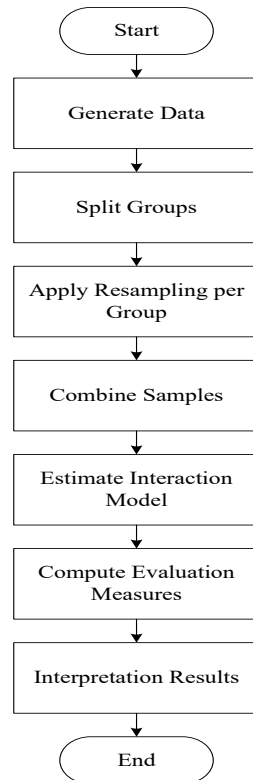


Figure 2. Group-Specific Resampling Procedure

2. 4. Performance Evaluation Metrics for Simulation Studies

The performance of the methods in this simulation study is evaluated using several evaluation measures, namely average bias, average standard error, standard error ratio, and coefficient of determination. The average bias is used to measure the accuracy of parameter estimation and is defined as follows.

$$\text{Bias} = \frac{1}{R} \sum_{r=1}^R (\hat{\theta}_r - \theta)$$

where $\hat{\theta}_r$ is the parameter estimate in the r -th replication, θ is the true population parameter, and R is the number of simulation replications. Furthermore, the average standard error is calculated as follows.

$$\overline{SE} = \frac{1}{R} \sum_{r=1}^R SE_r,$$

where SE_r is the standard error in the r -th replication. To evaluate the consistency of the standard error estimation, the standard error ratio is used, defined as follows.

$$RSE = \frac{\overline{SE}}{SD(\hat{\theta})},$$

where $SD(\hat{\theta})$ is the standard deviation of all parameter estimates obtained from the simulation results. In addition, the model's ability to explain the data's variation is evaluated using the coefficient of determination, defined as follows.

$$R_{u,adj}^2 = 1 - \frac{\frac{\sum_{i=1}^n (y_{ui} - \hat{f}_{ui})^2}{n - p - 1}}{\frac{\sum_{i=1}^n (y_{ui} - \bar{y}_u)^2}{n - 1}},$$

where

$R_{T,adj}^2$: total coefficient of determination

$R_{u,adj}^2$: coefficient of determination in the u -th model equation

y_{ui} : the i -th value of the endogenous variable

$\hat{f}_{ui}$: estimated function value for the i -th endogenous variable

$\bar{y}_u$: mean of the endogenous variable

p : number of model parameters

n : number of observations

Using these evaluation measures, the performance of each resampling method can be systematically compared to determine which produces the most stable standard error estimates.

3. RESULTS AND DISCUSSION

This section presents the results of the data analysis derived from the research data. The analysis evaluated the performance of the group-specific resampling method in a multigroup truncated-spline semiparametric path analysis model using a simulation study.

3.1. Simulation Study Result

The simulation study in this research was conducted to evaluate the performance of the group-specific resampling method in a multigroup truncated-spline semiparametric path analysis model with heterogeneous relationship structures across groups. The evaluation was performed using four resampling combinations: Bootstrap-Bootstrap (BB), Jackknife-Jackknife (JJ), Bootstrap-Jackknife (BJ), and Jackknife-Bootstrap (JB). Each combination was evaluated using several measures of estimation performance: average bias, average standard error, standard error ratio, and coefficient of determination. The results of the simulation study were used to identify the resampling method that provides the best inferential stability in a multigroup semiparametric model with linear and truncated spline relationship structures. The recapitulation results of the simulation study are presented in the following table.

Table 3. Simulation Study Results

Pattern		Average Bias	Average SE	Ratio SE	R^2
Group 1	Group 2				
Bootstrap	Bootstrap	0.13517	0.03799	2.79855	0.86522
Jackknife	Jackknife	0.13502	0.03226	2.37630	0.86444
Bootstrap	Jackknife	0.13504	0.03512	2.58662	0.86394
Jackknife	Bootstrap	0.13503	0.03512	2.58704	0.86664

Based on the simulation results in Table 3, all resampling method combinations produced relatively uniform average biases, ranging from 0.13502 to 0.13517. These results indicate that all resampling methods have nearly identical levels of estimation bias, suggesting that differences among resampling methods do not significantly affect the average bias in model parameter estimates.

Regarding the average standard error (Average SE), the jackknife–jackknife combination yielded the smallest value of 0.03226 compared to the other combinations. These results indicate that the jackknife method applied to both data sets produces more stable parameter estimates than other method combinations. Meanwhile, the BB combination yields the largest Average SE value of 0.03799, indicating that the resulting parameter estimates tend to exhibit higher variability.

Similar results were also observed for the SE Ratio. The jackknife–jackknife combination yielded the smallest SE Ratio value of 2.37630, while the bootstrap–bootstrap combination yielded the largest value of 2.79855. A smaller SE Ratio value indicates that the resulting standard error estimates are more consistent with the variation in parameter estimates, so the jackknife–jackknife method can be said to have more stable inferential performance in a multigroup semiparametric model.

Nevertheless, the jackknife–bootstrap combination yields the highest coefficient of determination (R^2) value of 0.86664 compared to the other combinations. These results indicate that this combination provides slightly better model explanatory power regarding data variability. However, the differences in R^2 values among the method combinations are relatively small, so overall, all resampling methods still produce nearly equivalent modeling quality.

Overall, the simulation results show that the jackknife method tends to produce more stable standard error estimates compared to the bootstrap method under conditions of a multigroup semiparametric model with a heterogeneous relationship structure. This is likely because the 5% delete jackknife approach is better at reducing local sensitivity in truncated spline models, particularly in data groups with knot-based nonlinear relationships.

3.2. Discussion

The results of the Ramsey RESET test on the empirical data indicate that most relationships are linear; however, some exhibit nonlinearity and are better modeled by a truncated linear spline with a single knot. The relationship between AI Use Attitudes (X_1) and Academic Adaptation (Y_1), as well as the relationship between Academic Adaptation (Y_1) and AI Literacy (Y_2), exhibits a linear pattern in Group 1, but in Group 2, it exhibits a truncated linear spline pattern with one knot point. These results indicate heterogeneity in relationship structure across groups, making a semiparametric approach relevant. This finding aligns with Lapenta's (2022) research, which explains that semiparametric models offer greater flexibility for capturing complex relationship patterns that are difficult to represent using parametric linear models, particularly in data that contain local variations and potential misspecifications.

Based on this empirical pattern, simulation data were generated using a combination of linear and truncated spline structures to realistically reflect the relationships observed in the actual data. The use of an empirical, structure-based simulation approach is important because it can yield more representative inferential evaluations than fully random simulations. Ferraccioli et al., state that empirical semiparametric structure-based simulations can improve the validity of inferential evaluations in models with heterogeneous, nonlinear relationships [17].

The results of the simulation study show that all combinations of resampling methods yield relatively uniform average biases, ranging from 0.13502 to 0.13517. This indicates that both bootstrap and jackknife methods exhibit relatively consistent parameter estimation capabilities in a multigroup semiparametric model. The very small difference in bias values indicates that changing the resampling method does not significantly alter the average deviation of the model's parameter estimates. These results support the findings of Cheng and Huang, who stated that both the bootstrap and the jackknife can produce relatively unbiased estimators in semiparametric and nonparametric models [18].

Differences in performance began to emerge in terms of the average standard error and the standard error ratio. The jackknife–jackknife combination yielded the smallest average standard

error of 0.03226 compared to the other combinations. This value indicates that the jackknife approach can produce more stable parameter estimates in a semiparametric multigroup model with a heterogeneous relationship structure. Additionally, the jackknife–jackknife combination also yields the smallest SE ratio of 2.37630, meaning the resulting inferences tend to be more consistent with respect to variations in parameter estimates.

This phenomenon is believed to occur because the 5% delete-jackknife approach is more effective at reducing the local sensitivity inherent in truncated spline models. In spline models, small changes in specific observations, particularly around knot points, can lead to significant shifts in parameter estimates. Consequently, the jackknife approach, which assesses model stability by partially removing observations, is better suited to the characteristics of spline-based nonlinear relationships. Mahmoud et al., explain that resampling-based inference in semiparametric and spline models tends to be more stable when robust methods are used to account for local data variations [19]. This finding is also supported by You et al., who state that the jackknife method produces more stable inferences in semiparametric models with complex data structures [20].

In addition, the results of this study also indicate that the resampling methods used tend to perform differently depending on the characteristics of the data relationship. Bootstrapping tends to be more suitable for linear, relatively stable relationships because it preserves the empirical distribution of the data through a with-replacement sampling process. In linear relationships, local variations in the data generally do not significantly affect parameter estimates, so the bootstrap can still produce sound inferences. Efron and Tibshirani explain that the bootstrap is highly effective for both parametric models and relatively stable relationships because it can flexibly represent the empirical distribution of the data [14].

Conversely, in nonlinear models based on truncated splines, the jackknife method tends to produce more stable estimates. This occurs because the jackknife method assesses the estimator's sensitivity by removing a subset of observations at each iteration, thereby reducing the influence of local variations around the knot points. In spline models, certain observations can significantly influence the formation of local curves, making inference methods that are more robust to local changes in the data even more necessary. Shao and Tu explain that the jackknife method is effective in evaluating the stability of estimators in nonparametric and semiparametric models that are sensitive to specific observations [16].

These findings are also supported by the study by Papalia et al., which compared the efficiency of the jackknife and double jackknife methods in path analysis [21]. That study showed that the jackknife-based approach produces more efficient parameter estimates and inferences than other resampling methods in path analysis models. These results indicate that the jackknife method has good capabilities in maintaining the stability of parameter estimates, particularly in models with complex relationship structures.

Consistent with these studies, this study's results show that the jackknife–jackknife combination yields the smallest average standard error and standard error ratio among the resampling combinations. This finding suggests that the jackknife approach is better at producing stable inferences in multigroup truncated-spline semiparametric path analysis models, particularly for data groups with spline-based nonlinear relationships.

Meanwhile, the BB combination yielded the largest Average SE and Ratio SE values compared to the other combinations. This indicates that the bootstrap method on both data groups resulted in relatively higher estimation variability. This occurs because bootstrap preserves all characteristics of the data's empirical distribution through with-replacement sampling, so that local variations in the spline relationship are carried over in each resampling iteration. In spline-based semiparametric models, this condition can increase the sensitivity of estimates to local changes in the data, particularly in groups with nonlinear relationships. Härdle et al., explain that bootstrapping in spline and generalized additive models tends to be more sensitive to the complexity of nonparametric functions and local variations in the data than other inference approaches [22].

Although the jackknife–jackknife combination yielded the best inferential stability, the jackknife–bootstrap combination produced the highest coefficient of determination (R^2) value of

0.86664. These results indicate that the group-specific resampling combination can still maintain good modeling quality despite data heterogeneity. This suggests that using different resampling methods for each group can be a more adaptive inference approach in multigroup models with heterogeneous relationship structures. MacKinnon et al., state that adaptive resampling strategies can improve the reliability of inference in models with heterogeneous and complex structures [23].

Overall, the results of this study indicate that the jackknife method is more stable than the bootstrap method in semiparametric path analysis using a multigroup truncated spline model, particularly for datasets with spline-based nonlinear relationships. These results reinforce the importance of using group-specific resampling approaches in semiparametric multigroup models, as the distinct characteristics of intergroup relationships can lead to differing inference requirements. Thus, this study makes a methodological contribution to the development of inference for semiparametric multigroup models, particularly regarding the selection of resampling methods that are better adapted to the heterogeneity of data relationship structures.

4. CONCLUSION

The results of the simulation study show that all combinations of resampling methods produce relative uniform average bias values; however, the jackknife-jackknife combination yields the smallest average standard error and standard error ratio, thereby providing better inferential stability compared to the other combinations. Conversely, the jackknife-bootstrap combination produces the highest coefficient of determination (R^2), although the difference is relatively small.

In general, the bootstrap method tends to be more suitable for relatively stable linear relationships, while the jackknife method is better suited to spline-based nonlinear relationships because it reduces local sensitivity to the data. Thus, this study shows that selecting resampling methods for semiparametric multigroup models should consider the characteristics of the relationships within each data group to produce more stable and accurate inferences.

This study has several limitations: the simulations were conducted on only two data groups with linear and truncated-spline relationships, and thus do not represent more complex multigroup conditions. Additionally, the jackknife method used was the 5% deletion approach. The 5% deletion method was chosen to maintain the stability of estimates in truncated spline models, which are sensitive to local changes in the data; however, this study has not evaluated variations in the proportion of observations deleted. This study also focuses on evaluating the performance of resampling methods in producing standard error estimates in multigroup semiparametric models; thus, it does not discuss other inferences, such as confidence intervals or resampling-based hypothesis testing, in greater depth. Furthermore, the context of Artificial Intelligence in learning is used solely as an empirical basis for creating simulation scenarios; thus, the research discussion emphasizes methodological aspects rather than substantive interpretations of learning variables.

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CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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